

# Unconstrained minimization

$$\text{minimize } f(x)$$

- $f$  convex, twice continuously differentiable (hence  $\text{dom } f$  open)
- we assume optimal value  $p^* = \inf_x f(x)$  is attained (and finite)

## unconstrained minimization methods

- produce sequence of points  $x^{(k)} \in \text{dom } f$ ,  $k = 0, 1, \dots$  with

$$f(x^{(k)}) \rightarrow p^*$$

- can be interpreted as iterative methods for solving optimality condition

$$\nabla f(x^*) = 0$$

## Initial point and sublevel set

algorithms in this chapter require a starting point  $x^{(0)}$  such that

- $x^{(0)} \in \mathbf{dom} f$
- sublevel set  $S = \{x \mid f(x) \leq f(x^{(0)})\}$  is closed

2nd condition is hard to verify, except when *all* sublevel sets are closed:

- equivalent to condition that  $\mathbf{epi} f$  is closed
- true if  $\mathbf{dom} f = \mathbf{R}^n$
- true if  $f(x) \rightarrow \infty$  as  $x \rightarrow \mathbf{bd} \mathbf{dom} f$

examples of differentiable functions with closed sublevel sets:

$$f(x) = \log\left(\sum_{i=1}^m \exp(a_i^T x + b_i)\right), \quad f(x) = -\sum_{i=1}^m \log(b_i - a_i^T x)$$